

ALGEBRA PROPERTIES

ARITHMETIC PROPERTIES

ASSOCIATIVE	$a(bc) = (ab)c$
COMMUTATIVE	$a + b = b + a$ and $ab = ba$
DISTRIBUTIVE	$a(b + c) = ab + ac$

ARITHMETIC OPERATIONS EXAMPLES

$$\begin{aligned}
 ab + ac &= a(b + c) & \frac{a}{b} - \frac{c}{d} &= \frac{ad - bc}{bd} \\
 a\left(\frac{b}{c}\right) &= \frac{ab}{c} & \frac{a-b}{c-d} &= \frac{b-a}{d-c} \\
 \left(\frac{a}{b}\right) &= \frac{a}{bc} & \frac{a+b}{c} &= \frac{a}{c} + \frac{b}{c} \\
 \frac{a}{\left(\frac{b}{c}\right)} &= \frac{ac}{b} & \frac{ab+ac}{a} &= b + c, a \neq 0 \\
 \frac{a}{b} + \frac{c}{d} &= \frac{ad + bc}{bd} & \left(\frac{a}{b}\right) &= \frac{ad}{bc} \\
 \left(\frac{c}{d}\right) &= \frac{bc}{ad} & &
 \end{aligned}$$

QUADRATIC EQUATION

For the equation $ax^2 + bx + c = 0$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

RADICAL PROPERTIES

$a, b \geq 0$ for even n

$$\begin{aligned}
 \sqrt[n]{a} &= a^{\frac{1}{n}} \\
 \sqrt[m]{\sqrt[n]{a}} &= \sqrt[mn]{a} \\
 \sqrt[n]{ab} &= \sqrt[n]{a}\sqrt[n]{b} \\
 \sqrt[n]{\frac{a}{b}} &= \frac{\sqrt[n]{a}}{\sqrt[n]{b}} \\
 \sqrt[n]{a^n} &= a, \text{ if } n \text{ is odd} \\
 \sqrt[n]{a^n} &= |a|, \text{ if } n \text{ is even}
 \end{aligned}$$

LOGARITHM PROPERTIES

if $y = \log_b x$ then $b^y = x$

$$\begin{aligned}
 \log_b b &= 1 \text{ and } \log_b 1 = 0 \\
 \log_b b^x &= x \\
 b^{\log_b x} &= x \\
 \log_a x &= \frac{\log_b x}{\log_b a} \\
 \log_b (x^r) &= r \log_b x \\
 \log_b (xy) &= \log_b x + \log_b y \\
 \log_b \left(\frac{x}{y}\right) &= \log_b x - \log_b y
 \end{aligned}$$

EXPONENT PROPERTIES

$$\begin{aligned}
 a^n a^m &= a^{n+m} \\
 (a^n)^m &= a^{nm} \\
 (ab)^n &= a^n b^n \\
 a^{-n} &= \frac{1}{a^n} \\
 \left(\frac{a}{b}\right)^{-n} &= \left(\frac{b}{a}\right)^n = \frac{b^n}{a^n} \\
 \frac{a^n}{a^m} &= a^{n-m} = \frac{1}{a^{m-n}} \\
 a^0 &= 1, a \neq 0 \\
 \left(\frac{a}{b}\right)^n &= \frac{a^n}{b^n} \\
 \frac{1}{a^{-n}} &= a^n \\
 a^{\frac{n}{m}} &= \left(a^{\frac{1}{m}}\right)^n = (a^n)^{\frac{1}{m}}
 \end{aligned}$$

PROPERTIES OF INEQUALITIES

If $a < b$ then $a + c < b + c$ and $a - c < b - c$
 If $a < b$ and $c > 0$ then $ac < bc$ and $a/c < b/c$
 If $a < b$ and $c < 0$ then $ac > bc$ and $a/c > b/c$

PROPERTIES OF COMPLEX NUMBERS

$$\begin{aligned}
 i &= \sqrt{-1} \\
 i^2 &= -1 \\
 \sqrt{-a} &= i\sqrt{a}, \quad a \geq 0 \\
 (a + bi) + (c + di) &= a + c + (b + d)i \\
 (a + bi) - (c + di) &= a - c + (b - d)i \\
 (a + bi)(c + di) &= ac - bd + (ad + bc)i \\
 (a + bi)(a - bi) &= a^2 + b^2 \\
 |a + bi| &= \sqrt{a^2 + b^2} \\
 \overline{(a + bi)} &= a - bi \\
 \overline{(a + bi)}(a + bi) &= |a + bi|^2 \\
 \frac{1}{(a + bi)} &= \frac{(a - bi)}{(a + bi)(a - bi)} = \frac{a - bi}{a^2 + b^2}
 \end{aligned}$$

COMMON FACTORING EXAMPLES

$$\begin{aligned}
 x^2 - a^2 &= (x + a)(x - a) \\
 x^2 + 2ax + a^2 &= (x + a)^2 \\
 x^2 - 2ax + a^2 &= (x - a)^2 \\
 x^2 + (a + b)x + ab &= (x + a)(x + b) \\
 x^3 + 3ax^2 + 3a^2x + a^3 &= (x + a)^3 \\
 x^3 + a^3 &= (x + a)(x^2 - ax + a^2) \\
 x^3 - a^3 &= (x - a)(x^2 + ax + a^2) \\
 x^{2n} - a^{2n} &= (x^n - a^n)(x^n + a^n)
 \end{aligned}$$

ABSOLUTE VALUE

$$\begin{aligned}
 |a| &= \begin{cases} a, & \text{if } a \geq 0 \\ -a, & \text{if } a < 0 \end{cases} \\
 |a| &= |-a| \\
 |a| &\geq 0 \\
 |ab| &= |a||b| \\
 \left|\frac{a}{b}\right| &= \frac{|a|}{|b|} \\
 |a + b| &\leq |a| + |b|
 \end{aligned}$$